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DEPARTMENT OF CIVIL ENGINEERING

EFFECTS OF AN OSCILLATING TORSIONAL COUPLE ON THE CONTACT SURFACES OF ELASTIC SPHERES

By

H. DERESIEWICZ

Office of Naval Research Project NR-064-388
Contract Nonr-266(09)
Technical Report No. 5
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Effects of an Oscillating Torsional Couple on the Contact Surfaces of Elastic Spheres

Introduction

The problem of two elastic bodies in Hertz contact subjected to a monotonically increasing torsional couple applied about the axis of symmetry of the system was first considered by Mindlin[1] who found that, in the absence of slip on the interface, the tangential traction rises to infinity on the elliptic boundary of contact. The torsional contact of two like spheres was reexamined by Lubkin[2] who made allowance for the occurrence of slip.

The present paper extends the Lubkin solution to the case in which the normal force is held constant while the torsional couple, after having reached a certain value, decreases, and to the case in which the torsional couple oscillates between fixed amplitudes. In the latter, the curve relating twisting moment to angle of twist forms a closed loop, the area of which represents the frictional energy dissipation per cycle. For small amplitudes, this energy loss is found to vary as the cube of the twisting moment. The solution is, strictly speaking, valid only for small amplitudes of the applied moment, because of an approximation made at the start. Practically, however, it is good over a large portion of the permissible range of moment-normal force ratios.

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(1) R. D. Mindlin, "Compliance of Elastic Bodies in Contact," J. Appl. Mech., Vol. 16, 1949, pp. 259-268.

(2) J. L. Lubkin, "The Torsion of Elastic Spheres in Contact," J. Appl. Mech., Vol. 18, 1951, pp. 183-187.

An approximate expression is obtained for the torsional compliance which depends upon the past history of loading. The initial value of the compliance during the first loading process is found to be the same as given in reference (1).

Summary of Previous Work

The Hertz problem for two like spheres[3] predicts a plane, circular contact surface of radius

$$a = (QNR)^{1/3} \tag{1}$$

where N is the normal force, R the radius of the spheres, and $Q = 3(1-\nu^2)/4E$, in which ν and E are Poisson's ratio and Young's modulus, respectively, of the material.

The distribution of normal traction on the contact surface is given by

$$\sigma = \frac{3N}{2\pi a^3}(a^2-\rho^2)^{1/2} \tag{2}$$

where ρ is the radial distance from the center of the contact surface.

If, now, an additional moment (M) is applied about the axis of normal contact, and if it is assumed that no slip occurs, symmetry considerations lead to the conclusion that no normal component of traction is induced and that the entire contact surface rotates as a rigid body with respect to a distant point of one of the spheres. The situation is thus reduced to a mixed boundary value problem in elasticity. With a system of cylindrical coordinates whose origin is at the center of the circle of contact and the

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(3) S. Timoshenko and J. N. Goodier, Theory of Elasticity, (McGraw-Hill Book Company, New York, 1951), p. 372.

z-axis along the axis of symmetry of the two spheres, the boundary conditions are: on the contact surface, the normal component of traction σ_z (zero) and the tangential component of displacement u_θ (proportional to the distance from the origin) are given. On the remainder of the surface of the sphere, approximated as plane, the three components of traction τ_{zr}, $\tau_{z\theta}$, σ_z (all zero) are given. Solution of this problem[1] gives the shearing stress $\tau_{z\theta}$ on the contact surface,

$$\tau \equiv \tau_{z\theta}\Big)_{z=0} = \frac{3M}{4\pi a^3}\rho\left(a^2-\rho^2\right)^{-1/2} \tag{3}$$

and the torsional compliance of a single body

$$c_\tau = \frac{d\beta}{dM} = \frac{3}{16\mu a^3} \tag{4}$$

where β is the angle of rotation of the contact surface with respect to a distant point in the body $z \geqslant 0$, and μ is the shear modulus of the material.

Since τ becomes infinite on the boundary of the contact circle, slip must result from an applied twisting moment, no matter how small. It may be expected to start at the edge of the contact surface where the singularity occurs, and, because of the radial symmetry of τ, to progress radially inward on an annulus. Further, it is assumed that, on the annulus of slip, $\tau = f\sigma$, where f is a constant coefficient of friction, and that it does not exceed this value elsewhere. On account of symmetry, the rigid rotation remains unaffected for the region over which no slip has occurred (the "adhered" portion). This gives rise to another mixed boundary value problem in elasticity. The normal component of traction σ_z (zero) and the tangential component of displacement u_θ (proportional to the distance from

the origin) are given on the adhered portion and the traction is given over the remainder of the boundary ($\tau = f\sigma$, $\tau_{zr} = \sigma_z = 0$ on the annulus of slip and all components of traction zero outside). Solution of this problem[2] yields the shear stress

$$\tau = \frac{3fN}{2\pi a^3}(a^2-\rho^2)^{1/2}, \qquad c \le \rho \le a$$

$$\tau = \frac{3fN}{2\pi a^3}(a^2-\rho^2)^{1/2}\left\{1 + \frac{2}{\pi}\left[\kappa^2 D(\kappa)F(\kappa',\varphi) - K(\kappa)E(\kappa',\varphi)\right]\right\}, \qquad \rho \le c \tag{5}$$

where c is the inner radius of the annulus, $\kappa' = c/a$, $\kappa = (1-\kappa'^2)^{1/2}$, $F(\kappa',\varphi)$ and $E(\kappa',\varphi)$ are incomplete elliptic integrals of the first and second kind, respectively, of modulus κ' and amplitude

$$\varphi = \arcsin \frac{1}{\kappa'}\sqrt{\frac{\kappa'^2-\rho^2/a^2}{1-\rho^2/a^2}}$$

and $\mathbb{D}(\kappa)$ is a tabulated[4] complete elliptic integral, of modulus κ,

$$\mathbb{D} = (\mathbb{K} - \mathbb{E})/\kappa^2 \tag{6}$$

Here, $\mathbb{K}$ and $\mathbb{E}$ are, respectively, the complete elliptic integrals of the first and second kind, of modulus κ. A cross-section of the distribution of τ is illustrated by curve aAO in Figure 1.

The relation between the applied moment and the inner radius of slip is given by the condition of equilibrium

$$M = 2\pi\int_0^a \tau\rho^2 d\rho \tag{7}$$

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(4) E. Jahnke and F. Emde, Tables of Functions, (Dover Publications, New York, 1945), pp. 73 and 83.

Hence,

$$M = \frac{fNa}{4\pi}\left\{ \frac{3\pi^2}{4} + \kappa'\kappa^2\left[6\mathbb{K} + (4\kappa'^2 - 3)\mathbb{D}\right] - 3\kappa\mathbb{K}\sin^{-1}\kappa' + 3\kappa^2\left[\mathbb{K}\int_0^{\pi/2} \frac{\sin^{-1}(\kappa'\sin\alpha)\,d\alpha}{(1-\kappa'^2\sin^2\alpha)^{3/2}} - \mathbb{D}\int_0^{\pi/2} \frac{\sin^{-1}(\kappa'\sin\alpha)\,d\alpha}{(1-\kappa'^2\sin^2\alpha)^{1/2}}\right]\right\} \tag{8}$$

Further, the relation between the angle of twist and the inner radius of slip is

$$\beta = \frac{3fN}{4\pi\mu a^2}\kappa^2\mathbb{D} \tag{9}$$

Equations (8) and (9) express, implicitly, the relation between the applied twisting moment and the resulting angle of twist. This is represented graphically by curve OPF in Figure 2. The value of the moment at which slip occurs over the entire contact surface, i.e., sliding is initiated, is $M/fNa = 3\pi/16$. This corresponds to an infinite value of the angle of twist.

Approximate Formulas for $M/fNa \ll 1$

In what follows it will be necessary to have explicit and reversible relations between each pair of the three quantities: inner radius of the slip annulus, twisting moment and angle of twist. This is manifestly

impossible for the exact solution [Eqs. (8) and (9)]. Let us, therefore, confine our efforts to the range of small M/fNa, i.e., $\kappa' = c/a \approx 1$.

Because of the divergence of the two integrals in Eq. (8) for $\kappa' = 1$, all efforts to express M of Eq. (8) as a polynomial in κ have been unsuccessful. For this reason the equation for M has been obtained in a different form. Starting with the following expression for τ[5],

$$\tau = \frac{3fN}{2\pi a^3}\left(a^2-\rho^2\right)^{1/2}, \qquad c \le \rho \le a$$

$$\tau = \frac{3fN}{\pi^2 a^4}\,\rho\left(c^2-\rho^2\right)^{1/2}\int_0^{\pi/2}\frac{a^2\kappa^2\sin^2\alpha/(a^2-\rho^2)}{\left[1-a^2\kappa^2\sin^2\alpha/(a^2-\rho^2)\right]\left(1-\kappa^2\sin^2\alpha\right)^{1/2}}\,d\alpha, \qquad \rho \le c \tag{10}$$

inserting it into the condition of equilibrium, Eq. (7), reversing the order of integration, and noting that[6]

$$\int_0^{\pi/2}\frac{\sin^2\alpha}{\sqrt{1-\kappa^2\sin^2\alpha}}\,d\alpha = \mathbf{D}$$

$$\int_0^{\pi/2}\sin^2\alpha\sqrt{1-\kappa^2\sin^2\alpha}\,d\alpha = \frac{2\kappa^2-1}{3\kappa^2}\mathbf{E} + \frac{\kappa'^2}{3\kappa^2}\mathbf{K}$$

we find

$$\frac{M}{fNa} = \frac{2}{\pi}\kappa'\kappa^2\mathbf{E} + \frac{6}{\pi}\kappa^3\int_0^{\pi/2}\sin^2\alpha\cos\alpha\sqrt{1-\kappa^2\sin^2\alpha}\,\tan^{-1}\left(\frac{\kappa}{\kappa'}\cos\alpha\right)d\alpha \tag{11}$$

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(5) In reference (2), insert Eq. (47) into Eq. (36). The a' is here replaced by c.

(6) L. Potin, Formules et Tables Numériques, (Gauthier-Villars, Paris, 1925), pp. 712,713.

This expression is easily expanded in a power series in κ with the result

$$\frac{M}{fNa} \cong \kappa^2\left(1 - \frac{3}{8}\kappa^2 - \frac{1}{64}\kappa^4 - \ldots\right) \tag{12}$$

Table I. compares the values of M/fNa, calculated by retaining two and three terms in the expansion, Eq. (12), with the exact values tabulated in reference (2).

Table I.

$\kappa' = c/a$	$\kappa^2 = 1 - \kappa'^2$	M/fNa				
		Exact	3-term Approx.	% Error	2-term Approx.	% Error
1	0	0	0	0	0	0
0.95	0.0975	0.0938	0.09392	0.1	0.09393	0.1
0.9	0.19	0.1764	0.17636	0	0.17646	0
0.85	0.2775	0.2482	0.24829	0	0.24862	0.2
0.8	0.36	0.3105	0.31067	0.1	0.31140	0.3
0.7	0.51	0.4082	0.41039	0.5	0.41246	1.0
0.6	0.64	0.4805	0.48230	0.4	0.48640	1.2
0.5	0.75	0.5286	0.53247	0.7	0.53906	2.0
0.4	0.84	0.5592	0.56613	1.2	0.57540	2.9
0.3	0.91	0.5769	0.58768	1.9	0.59946	3.9
0.2	0.96	0.5855	0.60058	2.6	0.61440	4.9
0	1	0.5890	0.60937	3.4	0.62500	6.1

A similar expansion of β in Eq. (9) yields

$$\frac{\mu a^2 \beta}{f N} \cong \frac{3}{16} \kappa^2 \left(1 + \frac{3}{8} \kappa^2 + \frac{15}{64} \kappa^4 + \ldots \right) \qquad (13)$$

but its use must be confined to small κ, since the expression in Eq. (9) diverges for $\kappa = 1$. Exact and approximate values of $\mu a^2 \beta / f N$ are compared in Table II.

Table II.

$\kappa' = c/a$	$\kappa^2 = 1 - \kappa'^2$	$\mu a^2 \beta / f N$				
		Exact	3-term Approx.	% Error	2-term Approx.	% Error
1	0	0	0	0	0	0
0.95	0.0975	0.0190	0.01899	0	0.01895	0.3
0.9	0.19	0.0385	0.03847	0.1	0.03817	1.0
0.85	0.2775	0.0586	0.05838	0.4	0.05744	2.0
0.8	0.36	0.0794	0.07866	0.9	0.07661	3.5
0.7	0.51	0.1234	0.11975	2.9	0.11392	7.7
0.6	0.64	0.1716	0.16527	3.7	0.15375	10.4

In view of the great complications arising from employing three or more terms in the expansions, Eqs. (12) and (13), we confine ourselves to two-term approximations. A short computation yields the following relations:

$$\frac{M}{f N a} = \frac{2}{3} \left[1 - \left(\frac{1 + 3 c^2/a^2}{4} \right)^2 \right] \qquad (14)$$

$$\frac{c}{a} = \frac{1}{\sqrt{3}} \sqrt{4 \left(1 - \frac{3}{2} \frac{M}{f N a} \right)^{1/2} - 1} \qquad (15)$$

$$\frac{\mu a^2 \beta}{fN} = \frac{3}{128}\left(1 - \frac{c^2}{a^2}\right)\left(11 - 3\frac{c^2}{a^2}\right) \tag{16}$$

$$\frac{\mu a^2 \beta}{fN} = \frac{1}{8}\left[1 - \left(1 - \frac{3}{2}\frac{M}{fNa}\right)^{1/2}\right]\left[3 - \left(1 - \frac{3}{2}\frac{M}{fNa}\right)^{1/2}\right] \tag{17}$$

Equation (17), which represents the explicit moment-twist relation, lead directly to the torsional compliance

$$C_\tau = \frac{3}{16\mu a^3}\left[2\left(1 - \frac{3}{2}\frac{M}{fNa}\right)^{-1/2} - 1\right] \tag{18}$$

It is seen that its initial value is the same as in Eq. (4).

Decreasing Torsional Moment(7)

Suppose, now, that, after having reached a value M^*, such that $0 < M/fNa < 3\pi/16$, the twisting moment M is reduced. If slip were prevented, the tangential component of traction τ would tend to negative infinity on the boundary $\rho = a$. This conclusion is reached from the solution of the appropriate boundary value problem in elasticity, as described in the section entitled "Summary of Previous Work". Hence, slip, opposite in sense to the initial slip, is presumed to start at $\rho = a$ and penetrate to a radius b, assumed, temporarily, to be of magnitude $c \leq b \leq a$. On the annulus $b \leq \rho \leq a$, $\tau = -f\sigma$, so that the change of tangential

(7) The procedure in this and the following sections was first employed in connection with tangential contact of two spheres. See R. D. Mindlin, W. P. Mason, T. F. Osmer and H. Deresiewicz, "Effects of an Oscillating Tangential Force on the Contact Surfaces of Elastic Spheres," Proc. 1st U. S. Nat'l. Cong. Appl. Mech. (1951), pp. 203-208.

traction is $-2f\sigma$. Since no additional slip occurs on the surface $\rho \leq b$, the change of displacement in that region must be that of a rigid body rotation. Thus, the change due to reduction of M presents a boundary value problem in elasticity identical in form with that solved in reference (2). Hence, by analogy with Eq. (5), the change in traction is

$$\tau_c = -\frac{3fN}{\pi a^3}(a^2-\rho^2)^{1/2}, \qquad b \leq \rho \leq a$$

$$\tau_c = -\frac{3fN}{\pi a^3}(a^2-\rho^2)^{1/2}\left\{1+\frac{2}{\pi}\left[\kappa_1^2 \mathbb{D}(\kappa_1)F(\kappa_1',\varphi_1) - \mathbb{K}(\kappa_1)E(\kappa_1',\varphi_1)\right]\right\}, \quad \rho \leq b \tag{19}$$

where κ_1, κ_1' and φ_1 are obtained from κ, κ' and φ, respectively, by replacing c by b. This distribution is illustrated by curve $aA'O$ in Figure 1.

The resultant traction accompanying a reduction in M is obtained by adding the initial traction, Eq. (5), and the change, Eq. (19), with the result:

$$\tau_R = -\frac{3fN}{2\pi a^3}(a^2-\rho^2)^{1/2}, \qquad b \leq \rho \leq a$$

$$\tau_R = -\frac{3fN}{2\pi a^3}(a^2-\rho^2)^{1/2}\left(1+\frac{4}{\pi}A_b\right), \quad c \leq \rho \leq b \tag{20}$$

$$\tau_R = -\frac{3fN}{2\pi a^3}(a^2-\rho^2)^{1/2}\left[1+\frac{2}{\pi}(2A_b - A_c)\right], \quad \rho \leq c$$

where A_b stands for the bracketed expression in Eq. (19) and A_c for a similar expression in Eq. (5). The traction profile is illustrated by curve $aDEO$ in Figure 1.

The condition of equilibrium, Eq. (7), yields, upon manipulation described in the section entitled "Approximate Formulas",

$$M = \frac{2fNa}{3}\left[1 - \left(\frac{1 + 3c^2/a^2}{4}\right)^2\right]$$

$$- \frac{4fNa}{3}\left[1 - \left(\frac{1 + 3b^2/a^2}{4}\right)^2\right] \qquad (21)$$

Noting, from Eq. (14), that the first term on the right-hand side of Eq. (21) is the initial torsional moment M^*, the inner radius of the annulus of counter-slip is

$$b = \frac{a}{\sqrt{3}}\sqrt{4\left(1 - \frac{3}{2}\,\frac{M^* - M}{2fNa}\right)^{1/2} - 1} \qquad (22)$$

Thus, as long as $-M^* \leq M \leq M^*$, the assumption $c \leq b \leq a$ is valid. When $M = -M^*$, i.e., when the twisting moment is fully reversed, $b = c$, i.e., counter-slip has penetrated to the depth of initial slip. At the same time, Eqs. (20) reduce to Eqs. (5) with signs reversed. The traction is then distributed just as the initial traction at $M = M^*$ was, but with opposite sense. The situation is the same as if no positive M had ever been applied, but only a negative M of magnitude M^*.

The associated rotation of the adhered portion is found by a similar process of superposition. First, the change in angle of twist is obtained by multiplying Eq. (16) by -2 and replacing c by b as given in Eq. (22). The initial twist is given by Eq. (16) in which the value of c is found from Eq. (15) wherein M is replaced by M^*. The resultant angle of twist, given by the sum of the two, is given by

$$\frac{\mu a^2 \beta_d}{fN} = \frac{3}{128}\left(1 - \frac{c^2}{a^2}\right)\left(11 - 3\frac{c^2}{a^2}\right) - \frac{3}{64}\left(1 - \frac{b^2}{a^2}\right)\left(11 - 3\frac{b^2}{a^2}\right)$$

$$= \left(1 - \frac{3}{2}\,\frac{M^* - M}{2fNa}\right)^{1/2} - \frac{1}{2}\left[1 + \left(1 - \frac{3}{2}\,\frac{M^*}{fNa}\right)^{1/2}\right] - \frac{3}{16}\,\frac{M}{fNa} \qquad (23)$$

The relation between twisting-moment and angle of twist is illustrated by the full lines $OPRS$ in Figure 2.

The compliance for unloading is

$$c_\tau = \frac{3}{16\mu a^3}\left[2\left(1 - \frac{3}{2}\,\frac{M^* - M}{2fNa}\right)^{-1/2} - 1 \right] \tag{24}$$

Thus, the initial compliance on unloading [$M = M^*$ in Eq. (24)] is the same as the initial compliance on loading [$M = 0$ in Eq. (18)]. That is, in Figure 2, the slope of PR at P is the same as the slope of OP at O.

When M has been reduced from M^* to zero, there is a permanent set given by OR in Figure 4, the magnitude of which is obtained by setting $M = O$ in Eq. (23). The accompanying traction is not zero, but is a self-equilibrating distribution obtained by setting

$$b = \frac{a}{\sqrt{3}}\sqrt{4\left(1 - \frac{3}{4}\,\frac{M^*}{fNa}\right)^{1/2} - 1}$$

in Eqs. (20).

When M has been reduced to $-M^*$, the twist has reached the negative of the twist at $M = M^*$ (i.e., the abscissa of S in Figure 2 is the negative of the abscissa of P) and the compliance is identical with that of curve OP at $M = M^*$. Hence, the unloading curve PRS is tangent, at S , to the central perversion, OS , of the loading curve OP.

Oscillating Torsional Moment

In the preceding section, the entire situation at $M = -M^*$ is identical with that at $M = M^*$, except for reversal of sign. Hence, a subsequent increase of M from $-M^*$ to M^* will be accompanied by the same events as

occurred during the decrease from M^* to $-M^*$, except for reversal of sign. Thus, in starting along SU, Figure 3, the compliance at S is the same as the compliance of PR at P. Slip again starts at $\rho = a$ in the same sense as occurred along path OP. At an intermediate point of SU the traction will be like $aDEO$, Figure 1, with sign reversed. When the twisting moment once more reaches M^*, b will again have penetrated to c and the traction will be exactly aAO, Figure 1.

The twist along path SUP is

$$\beta_i = -\beta_d(-M)$$

so that

$$\frac{\mu a^2 \beta_i}{fN} = -\left(1 - \frac{3}{2}\,\frac{M^* + M}{2fNa}\right)^{1/2} + \frac{1}{2}\left[1 + \left(1 - \frac{3}{2}\,\frac{M^*}{fNa}\right)^{1/2}\right] - \frac{3}{16}\,\frac{M}{fNa} \tag{25}$$

Hence, when $M = M^*$, the terminal points P of SUP and P of OP (Figure 3) are identical, as may be seen by comparing Eq. (25) with Eq. (17) for that value of M.

Subsequent diminution of M will then produce a repetition of the events accompanying the first diminution.

It is now evident that, during oscillation of M between $\pm M^*$, with N maintained constant, the torque-twist curve traverses the loop $PRSUP$, Figure 3.

The area enclosed in the loop gives the frictional energy loss per cycle:

$$F = \int_{-M^*}^{M^*} (\beta_d - \beta_i)\, dM$$

$$= \frac{2f^2N^2}{\mu a}\left\{ \frac{8}{9}\left[1 - \left(1 - \frac{3}{2}\,\frac{M^*}{fNa}\right)^{3/2}\right] - \frac{M^*}{fNa}\left[1 + \left(1 - \frac{3}{2}\,\frac{M^*}{fNa}\right)^{1/2}\right]\right\} \tag{26}$$

For $M^*/fNa \ll 1$, Eq. (26) reduces to

$$F = \frac{M^{*3}}{18\mu a^4 f N} \qquad (27)$$

i.e., the energy loss per cycle varies as the cube of the maximum torsional moment. It is interesting to note the close correspondence between oscillating torsional contact and oscillating tangential contact of two spheres. In the latter case, for amplitudes of tangential force small in comparison with fN, the energy loss per cycle is equal to[7]

$$F = \frac{(2-\nu)T^{*3}}{36\mu a f N} \qquad (28)$$

where T^* is the maximum tangential force.

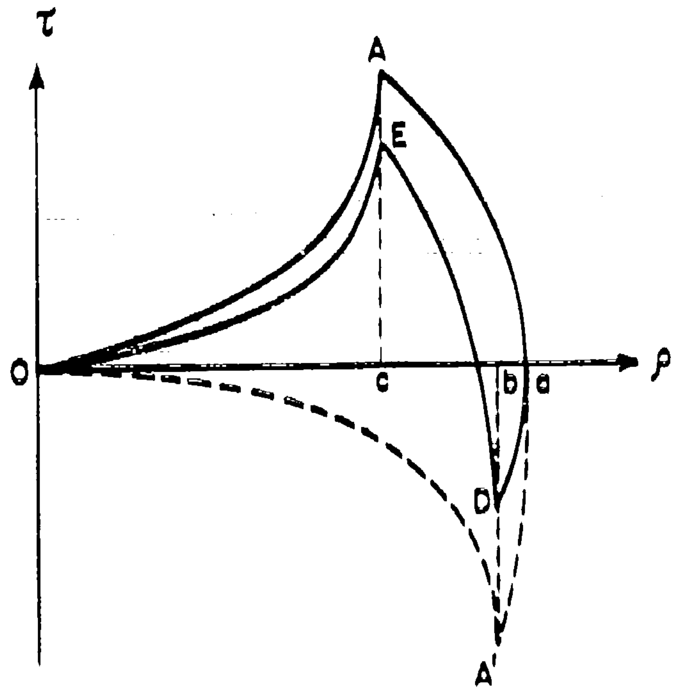

Fig. 1.

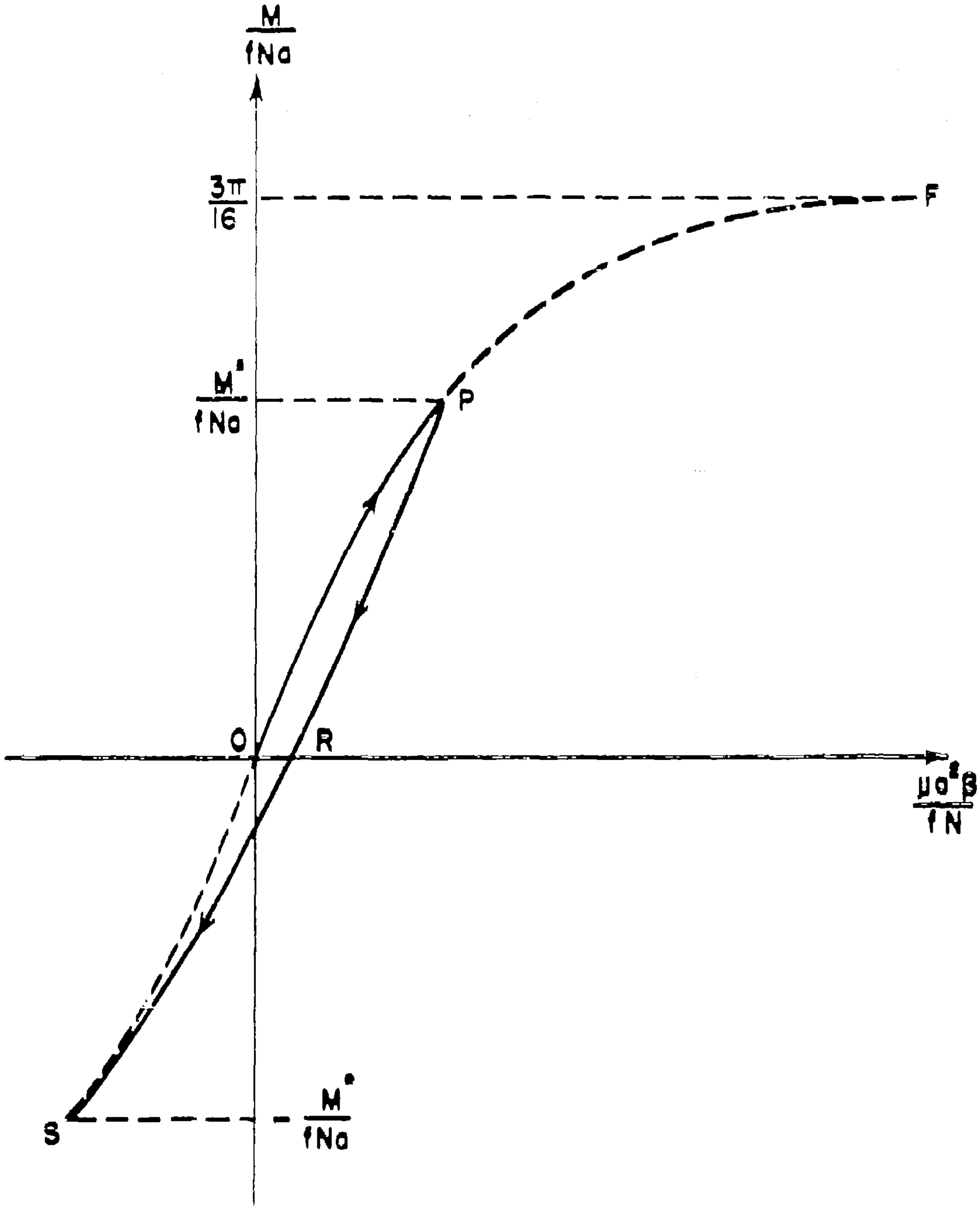

Fig. 2.

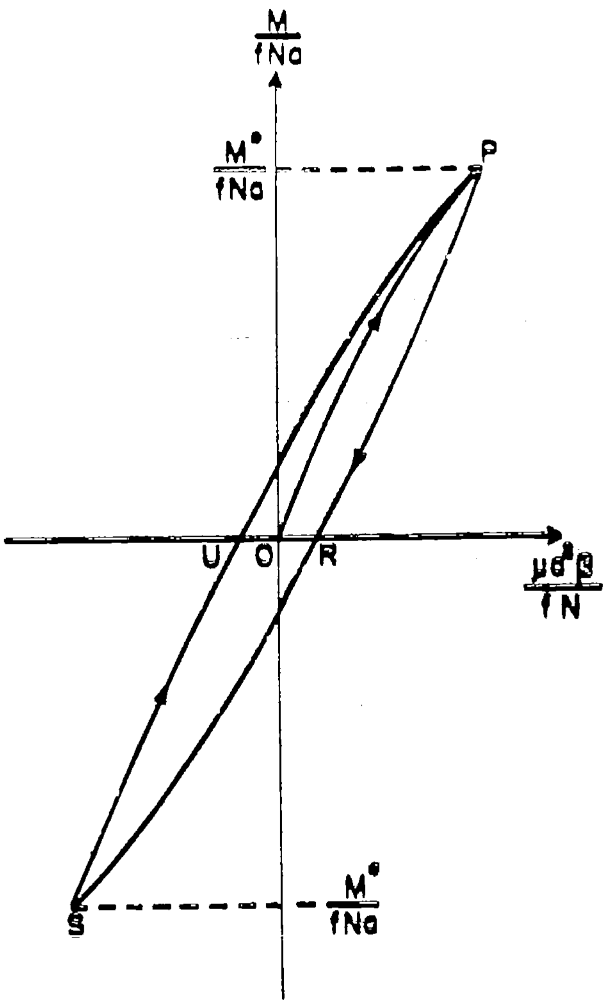

Fig. 3.

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